Glossary of Terms for Multilevel Models

1. **–2LL** – Known as the model deviance, the –2LL is the sum of the log-likelihood of the observations taken across all observations. As an indicator of model misfit (i.e., badness of fit), lower values are better. It is the basis for AIC and BIC, which are information criteria that penalize for parameters and sample size (lower is still better, though).
2. **Clustered Sample** – The sampling of subjects from pre-existing groups in the absence of repeated-measurement, such as children in schools, patients in physicians, siblings in families, people in neighborhoods, etc.
3. **Compound Symmetry** – A covariance structure that requires equal variances and equal covariances. It is assumed by the univariate repeated-measures ANOVA model.
4. **Conflated Effect** – See smushed effect.
5. **Centering** – Subtraction or addition of a constant that relocates the meaning of a 0 value for a predictor variable, used to aid the interpretation of fixed and random intercepts, as well as of fixed slopes of predictor variables that are also part of interaction slopes.
6. **Convergence Effect** – See smushed effect.
7. **Cross-level Interaction** – A model-estimated constant slope for the multiplicative combination of two predictor variables measured for different dimensions of sampling.
8. **Data Scale** – The scale of measurement of the observed outcome and serves to assist with interpretation. The data scale generally only applies to conditionally non-normal distributions with data-scale estimated obtained by inverse-linking the model-scale estimates.
9. **Dependency** – The presence of correlation (i.e., non-independence) among residuals from the same higher-level sampling unit (e.g., people in longitudinal data or clusters in clustered data). Dependency is captured in the multilevel model using random effects.
10. **Deviance Difference Test** – see Likelihood Ratio Test (LRT)
11. **Fixed Effect** – A model-estimated constant that is used in linear combination with predictor variable values to create a predicted (expected) outcome for each observation. Includes fixed intercept and fixed slopes. All fixed effects are assumed to apply to all individuals in the sample as they represent the average effect on the outcome. In linear regression, fixed effects were denoted by beta (β). By contrast, in the multilevel model, the betas (β) serve as placeholders and the regression slopes are denoted by gamma (γ).
12. **Fixed Intercept** – A model-estimated constant that provides the predicted (expected) outcome value when all predictor variables are 0.
13. **Fixed Slope** – A model-estimated constant that provides the predicted (expected) difference in an outcome variable per one-unit difference in a predictor variable value.
14. **G-matrix** – The covariance matrix that holds random effect variances on the diagonal and covariances between random effects on the off-diagonals.
15. **Grand-mean Centering** – Subtraction of a constant (that may or may not be the mean) from all predictor variable values. In the context of multilevel models, grand-mean centering retains the between- and within-variance of the original uncentered predictor.
16. **Group-mean Centering** – See Variable centering.
17. **Intra-class Correlation (ICC)** – For two-level multilevel models, the ICC quantifies the proportion of variability in the outcome that is level-2, between-person for longitudinal

samples or level-2, between-clusters for clustered samples. The ICC ranges from 0 to 1, with values closer to 1 indicating more outcome variability at level 2.

18. **Link Function** – A transformation of the predicted mean outcome from the data (outcome) scale of measurement onto a continuous, linear, unbounded model scale that is symmetric around 0. For conditionally normally distributed outcomes, the data scale and the model scale are synonymous.
19. **Likelihood Ratio Test (LRT)** – A comparison of the difference in –2LL between two nested models. The difference in –2LL follows a chi-square distribution with degrees of freedom equal to the difference in the number estimated parameters. If using REML, the number of parameters only includes the parameters estimated in the Model for the Variance. If using ML, the number of parameters includes all parameters estimated in the Model for the Variance and Model for the Means. If using REML, the LRT can only be used if the comparison models have identical fixed effects.
20. **Longitudinal Sample** – The repeated sampling of observations from the same unit (e.g., person or group), such as over time, over conditions, over stimulus-specific trials, etc.
21. **Model for the Means** – Within the regression equation, it is the linear combination of fixed effects (betas; βs) and predictor variable values (Xs) used to predict the outcome value (Y) at a given observation. This model is also known as the structural model.
22. **Model for the Variance** –Within the regression equation, it is the random effect (Us) and residual (es) values. This model captures dependency (i.e., non-independence); that is, how residuals and random effects are distributed and related across observations. This model is also known as the stochastic model.
23. **Model Scale** – The fixed- and random-effect estimates on the link-transformed scale. This is the scale on which statistical inferences (i.e., p-values) are obtained.
24. **Nested Model** – A nested model is one that can be specified as a comparison model by constraining some of its estimated parameters (e.g., fixed effects, random effects) to be 0.
25. **Parsimony** – Reproducing the observed data with as few estimated parameters as possible.
26. **Person-mean Centering** – See Variable centering.
27. **Random Effect** – Latent (unobserved) variables with model-estimated variances that are used in a linear combination with predictor variables and fixed effects to create an expected outcome for each observation. In the multilevel model, random effects represent deviations from the model-estimated average effect of the fixed intercept and fixed slopes. Notationally, it is denoted by Us. The variance of a random effect is denoted τ^2, which will have a subscript indicating which random effect that variance is specific to.
28. **Random Intercept** – The difference (or deviation) between the fixed intercept and the intercept specific to a higher-level sampling unit (e.g., person in longitudinal data or cluster in clustered data). In a two-level multilevel model, it is denoted by $U_{0,i}$ for longitudinal data or $U_{0,j}$ for clustered data, in which the 0 subscript indicates the fixed effect it is attached to (i.e., $\beta_{0,i}$ or $\beta_{0,j}$) and the i or j subscript indicates that it is specific to individual i or group j the random intercept is specific to. The random intercept variance is denoted $\tau^2_{U_0}$.

29. **Random Slope** – The difference (or deviation) between a fixed slope and the slope specific to a higher-level sampling unit (e.g., person in longitudinal sample or cluster in clustered sample). In a two-level multilevel model, it is denoted by $U_{1,i}$ for longitudinal sample or $U_{1,j}$ for clustered sample, in which the subscript 1 indicates the fixed slope it is attached to (i.e., $\beta_{1,i}$ or $\beta_{1,j}$; note that it could also be $U_{2,i}$ and $\beta_{2,i}$ or $U_{4,i}$ and $\beta_{4,j}$ depending on the fixed effect the random slope is associated with) and the i or j subscript indicates that it is specific to individual i or group j the random slope is specific to. A random slope variance is denoted $\tau^2_{U_1}$ (or $\tau^2_{U_2}$ and so on).
30. **Residual** – The difference (or deviation) between the outcome predicted by the model's fixed effects and random effects and the actual outcome. In a two-level multilevel model, the level-1 residual value represents the deviation of an observation from the person's mean (in longitudinal samples) or cluster mean (in clustered samples). The level-1 residual variance is denoted by σ^2_e.
31. **R-matrix** – The covariance matrix that holds residual variances on the diagonal and covariances between residuals on the off-diagonals. Once all random effects are properly included in the model, the assumption is that residual values are no longer correlated (i.e., independent).
32. $\boldsymbol{\sigma^2_e}$ – Pronounced sigma-squared. It indicates the level-1 residual variance as indicated by the subscript e.
33. **Single-level Interaction** – A model-estimated constant slope for the multiplicative combination of two predictor variables measured for the same dimension of sampling.
34. **Smushed Effect** – The unintended blending of a predictor variable's slopes across multiple levels of sampling, by which the distinct predictor slopes are constrained equal. A common consequence of failing to partition the between- and within-variance inherent to lower-level predictors.
35. $\boldsymbol{\tau^2}$ – Pronounced as tau-squared. It indicates a higher-level (e.g., level-2) random-effect variance. The subscript will identify which random effect (i.e., intercept or slope) the variance is specific to.
36. **Time-invariant Predictor** – A variable is this measured only one time during the study period or a variable that is constant (or changes minimally) over the entirety of the study period. Examples include race and chronological age at study entry or in a single-year study. In a two-level longitudinal multilevel model, time-invariant predictors are at the level-2, person-level and represent between-person, inter-individual differences.
37. **Time-varying Predictor** – A variable that is measured repeatedly at the same time as the outcome. Its values are expected to change within a person over the study period. Examples include income, self-efficacy, and stress. In a two-level longitudinal multilevel model, time-varying predictors are at the level-1, occasion-level and represent within-person, intra-individual variation.
38. **Unstructured** – A covariance structure that estimates all variances and all covariances. If estimable, this structure will perfectly reproduce your data and, therefore, is not a statistical model. This covariance structure is assumed by the multivariate repeated-

measures ANOVA model. It can serve as the comparison model when looking to estimate a more parsimonious model.

39. **Variable Centering** – Subtraction or addition of a variable that not only relocates the meaning of a 0 value for a predictor variable, but also removes higher-level unit differences (such as due to mean differences across the higher-level units).
40. **V-matrix** – The covariance matrix that combines the **G**- and **R**-matrices. The **V**-matrix holds total variance of the outcome on the diagonal and covariances between outcomes on the off diagonal. The **V**-matrix is the matrix on which the statistical analysis is performed.